MATH 90 – CHAPTER 2 Name: .

2.1 Solving Equations

Need To Know

- Check a solution to an equation
- Understand Addition property of equality
- Understand Multiplication property of equality
- Use them to solve equations

Equations

Definition – The _____ to an equation is the set of all numbers that can replace the variable and make the equation a true statement.

Examples:

Is 4 a solution to $2x + 3 = 7$? Is 4/3 a solution to $8 = 3x + 4$?

Equivalent Equations

Definition – _____ are two or more equations with the same solution.

Example:

$2(x + 3) = 16$

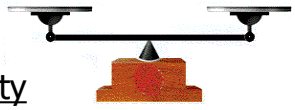
Observation:
The _____ is a very useful equivalent equation.

HOW?
How can we make equivalent equation.

Addition Property of Equality

Any change must maintain the balance.

$x - 7 = 10$



Addition Property of Equality

_____

In Words:
You can make equivalent equations
by adding or subtracting the same number to both sides of the equation.

Solve Equations

Example:

$x + 3 = 9$

Solving Equations

Goal:

- _____.

How:

- ____ equivalent _____ _____ all the operations associated with the unknown.
- Show your thinking by writing the algebra step on _____ of the equal sign.

Solve Equations

Solve for x

$x - 4 = 12$

Solve for w

$-3.5 + w = 8.2$

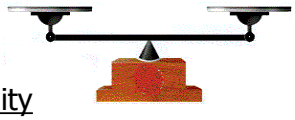
Solve for b

$b + \frac{2}{3} = -\frac{1}{6}$

Multiplication Property of Equality

Any change must maintain the balance.

$2x = 16$



Multiplication Property of Equality

_____

In Words:
You can make equivalent equations by
multiplying or dividing the same non-zero number to both sides of the equation

Multiplication Property of Equality

Example: Solve each.

$5x = -20$ $-3r = 27$ $-\frac{5x}{6} = 3$

2.2 Solving Harder Equations

Need To Know

- Solve equations that require both properties.
- Solve equations with like terms on both sides.
- Simplifying equations with fractions.

Solve Equations

What's the Goal?

Solve for x

$7 + 1 = 2x - 6x$

Steps to Solve

Simplify

1.
2.
3. _____

Use Add Property to

1. Get variable on one side
2. Get constant terms on the other side

Use Multiplication Property

1. Get unknown by itself

Check

Solve Multi-step Equations

Solve for x

$4x + 3 = -13$

Steps to Solve

Simplify

1.
2.
3. Clear like terms

Use Add Property to

1. Get variable on one side
2. Get constant terms on the other side

Use Multiplication Property

1. Get unknown by itself

Check

Solve Multi-step Equations

Solve

$3x + 4 = -15$

Steps to Solve

Simplify

1.
2.
3. Clear like terms

Use Add Property to

1. Get variable on one side
2. Get constant terms on the other side

Use Multiplication Property

1. Get unknown by itself

Check

Solve Multi-step Equations

Solve

$2(x - 3) + 3 = 9$

Steps to Solve

Simplify

1. _____
2.
3. _____

Use Add Property to

1. Get variable on one side
2. Get constant terms on the other side

Use Multiplication Property

1. Get unknown by itself

Check

Solve Multi-step Equations

Solve for z:

$5z + 6 = 3z - 6$

Steps to Solve

Simplify

1. Clear parentheses
2.
3. Clear like terms

Use Add Property to

1. Get variable on one side
2. Get constant terms on the other side

Use Multiplication Property

1. Get unknown by itself

Check

Solve Multi-step Equations

Solve

$7(x - 3) + 5 = 4(3x - 2) - 8$

Steps to Solve

Simplify

1. Clear parentheses
2.
3. Clear like terms

Use Add Property to

1. Get variable on one side
2. Get constant terms on the other side

Use Multiplication Property

1. Get unknown by itself

Check

Solve Equations w/ Fractions

Solve for x:

$$\frac{1}{3}x+\frac{2}{5}=\frac{4}{15}+\frac{3}{5}x-\frac{2}{3}$$

Steps to Solve

Simplify

1. Clear parentheses
2. _____
3. Clear like terms

Use Add Property to

1. Get variable on one side
2. Get constant terms on the other side

Use Multiplication Property

1. Get unknown by itself

Check

2.3 Formulas

Need To Know

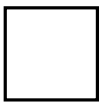
- Basic formulas
- Evaluating formulas
- Solving formulas
- Cast Away

Formulas to Memorize

Square

Area = s^2

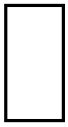
Perimeter = 4s



Rectangle

Area = LW

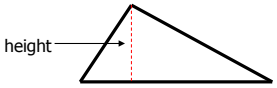
Perimeter = 2L + 2W



Triangle

Area = ½ bh

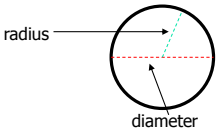
Perimeter = a + b + c



Circle

Area = πr^2

Circumference = $2\pi r$



Evaluating Formulas

Power P, in watts, of an electrical appliance is found by P = V • I, where I is the current, in amps, and V is the voltage, measured in volts. If a kitchen requires 30 amps and there is 115 volts in the house, what is the wattage in the kitchen?

A farmer has 76 feet of fencing. He plans to make a pig pen that is 10 feet wide. How long will it be?

Evaluating Formulas

When 400 mg of the painkiller ibuprofen is swallowed, the number of n milligrams in the bloodstream t hours later (for $0 \leq t \leq 6$) is estimated by

$$n = 0.5t^4 + 3.45t^3 - 96.65t^2 + 37.7t$$

How many milligrams of ibuprofen remain in the blood 1 hr after 400 mg has been swallowed?

Solving Formulas

Sometimes formulas need to be rearranged.
We must solve for a different variable in the formula.
We use algebra to isolate that variable.
In other words, we undo all the operations associated with that variable to get it by itself.

Solve for a:	Solve for L:	Solve for t:
c = a + b	A = LW	I = Prt

Practice

Solve for L
$P = 2W + 2L$

Solve for y
$2x - 5y = 35$

Practice

Solve for x
$S = rx + sx$

Solve for y
$$H = \frac{a + y - k}{4}$$

end

2.4 Percent

Need To Know

- Percentage Notation
- Percent Equations
- Percent Word Problems

Percentage

Recall percent means out of 100.

Percent is a concept. It must be translated into a numerical value.

Percent Notation: n% means $\frac{n}{100}$ *or* $n \cdot \frac{1}{100}$ *or* $n \cdot 0.01$

Percent	Decimal	Fraction
50%	0.50	½
35%		
	0.375	
		$\frac{7}{5}$

Solve Percent Equations

Key Words:

"What"

"is" or "was"

%

What number is 75% of 40?

What percent of 20 is 14?

12% of what number is 240?

Percent Translation

Definition – A **verbal model** is a pseudo sentence that describes the mathematics of a common situation.

Many percent problems follow this verbal model:

"A percent of the whole is the part."

Example: This ice cream is 150 calories. 90 calories are from fat which is 60%.

Percent = 60%
Whole = 150 calories
Part = 90 calories

A percent of the whole is the part.
60% of 150 calories = 90 calories

Percent Word Problems

If a serving of ice cream is 65 grams and the container says that each serving has 11 grams of sugar, what percent of the ice cream is sugar?

Percent Word Problems

Glenn takes out a student loan for $2400.
After a year, Glenn decides to pay off the interest, which is 7% of $2400. How much will he pay?

Percent Word Problems

Mike wants to leave a 12% tip on a meal that cost $35.50.
How much is the tip?

Percent Word Problems

Sierra left a 15% for her meal.
The final amount was $33.58.
What was the cost of the meal before the tip?

2.5 Problem Solving

Need To Know

- Guide lines to solve word problems
- How to become familiar with a problem
- Apply

Guide Lines to Solve Problems

Five Steps for Problems Solving

1. Familiarize myself with the problem.
2. Translate to mathematics (i.e. an equation).
3. Carry out the mathematics (i.e. solve).
4. Check your answer in the original problem.
5. State your answer clearly.

Familiarize Yourself w/ Problem

1. Read and reread the problem, visualize it, read it aloud and understand key words.
2. List the given information and question(s) to be answered
3. Choose a variable letter and specify what the variable represents (**REQUIRED**)
4. Find more information, look up formulas or do research need to start.
5. Use the other TOOLS.

Apply

Subtracting 5 from twice a number is 11.
Find the number.

Steps
1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools
1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model

Apply

In Cranston, taxis charge $4 plus 90¢ per mile for an airport pick-up.
How far can Ralph get for $17.50?

Steps
1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools
1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model

Apply

Steps
1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

The perimeter of a triangle is 195 mm. If the lengths of the sides are consecutive odd integers, find the sides.

Tools
1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model

Apply

Steps
1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Milton borrowed money from his friend at a rate of 10% simple interest. One year later he paid $7194 to clear the loan. How much did he borrow?

Tools
1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model

2.6 Linear Inequality

Need To Know

- Solving Inequalities
- Graphing Inequalities
- Set-Builder and Interval Notation
- The Add Property for Inequalities
- The Mult. Property for Inequalities
- Using the Two Properties Together

Solving Inequalities

An inequality is any math sentence with $<$, $\leq$, $>$, $\geq$, $\neq$

Examples:

$3x + 2 > 7$, $c \leq 7$, and $4x - 6 \neq 3$.

A **solution** is any value that make the inequality true.
The set of all solutions is called the **solution set.**

Examples:

Determine if 5 solves: $3x + 2 > 7$.

Set-Builder and Interval Notation

Endpoints that equal graph with] or [
Endpoints that are not equal graph with) or (

Graph each:

$x \leq -2$

$x < 5$

$-3 < x \leq 1$

Each graph is a set of infinite numbers.

Set-Builder Notation

Set-builder Notation
Explains the set with a formula. {x | formula for x}

Write the set of each graph in set-builder notation

$x \leq -2$

$x < 5$

$-3 < x \leq 1$

Interval Notation for Sets

An **interval** expresses a set of numbers. They are written:

- Small to BIG
- Parentheses (,) means to exclude the end points from the set
- Brackets [,] means to include the end points from the set.

If *a* and *b* are real numbers such that $a < b$:

The **open interval (*a, b*)** is the set

The **closed interval [*a, b*]** is the set

The **half-open interval (*a, b*]** is the set

The **interval [*a*, ∞)** is the set

The **interval (-∞, *a*)** is the set

Practice: Set Builder and Intervals

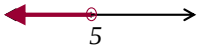
Graph and write each in set-builder:

1. (-2, 4]
2. [3, ∞)

Graph and write in interval notation:

3. $\{x \mid 1 < x \leq 7\}$

Write each in set-builder and interval notation:

4. 

5. 

Solving Inequalities

Solving inequalities is the same as solving equations except for one special situation.

Addition Property of Inequality

If $A < B$,

Then $A + C < B + C$

In Words:
You can make equivalent inequalities
by adding or subtracting the same number to both sides of the inequality

Solving Inequalities

Solve: $3x - 7 \geq 2x + 3$ and graph solution.

Idea of Multiplying Inequalities

Consider multiplying both sides by a **positive**:		Consider multiplying both sides by a **negative**:	
1)	$2 < 6$	1)	$2 < 6$
2)	$-2 < 6$	2)	$-2 < 6$
3)	$-2 > -6$	3)	$-2 > -6$

Solving Inequalities

Multiplication Property of Inequality

1) If $A < B$,
Then $AC < BC$ if C is positive.

Multiplying or dividing a positive to both sides of the inequality will keep the same solution set.

2) If $A < B$,
Then $AC > BC$ if C is negative.

Multiplying or dividing a negative to both sides of the inequality requires switching the inequality to keep the same solution set.

Solving Inequalities

Solve and graph:
$-5a \leq 20$

Solve and graph:
$3x + 4 > 16$

Solving Inequalities

Solve and graph:
$2(3m - 1) + 5 \geq 8m - 7$

Solving Inequalities

Solve and graph:

$$\frac{1}{3}y - \frac{1}{2} \leq \frac{5}{6} + \frac{1}{2}$$

2.7 Solving Inequality Applications

Need To Know

- Translating Inequalities
- Solving Inequality Problems

Translating Inequalities

Important Words	Sample Sentence	Translation
is at least	Max is at least 5 years old.	
are at most	There are at most 6 people in the car.	
cannot exceed	Total weight in the elevator cannot exceed 2000 pounds.	
must exceed	The speed must exceed 15 mph.	
is between	Heather's income is between $23,000 and $35,000.	$23{,}000 < h < 35{,}000$
no more than	Bing weighs no more than 90 pounds.	$w \leq 90$
no less than	Saul would accept no less than $5000 for his used car.	$t \geq 5000$

Translating Inequalities

1. A number is less than 10.
2. A number is greater than or equal to 4.
3. The temperature is at most
4. The average credit-card debt is more than $8000.
5. To rent a car, a driver must have a minimum of 5 yr driving experience.
6. Tania earns between $12 and $15 an hour.
7. Leslie's test score was at least 85.
8. Wind speeds were greater than 50 mph.
9. The costs of production of that software cannot exceed $12,500.
10. The cost of gasoline was at most $4 per gallon.

Guide Lines for Problem Solving

Five Steps for Problems Solving

1. Familiarize myself with the problem.
2. Translate to mathematics (i.e. an equation).
3. Carry out the mathematics (i.e. solve).
4. Check your answer in the original problem.
5. State your answer clearly.

Application

Rod's quiz grades are 73, 75, 89, and 91.
What scores on a fifth quiz will make his average quiz grade at least 85?

Steps
1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools
1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model

Application

Steps
1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tom's construction work can be paid to him in two ways:

Plan A: $300 plus $9 per hour or

Plan B: Straight $12.50 per hour.

If the job takes *n* hours,
what is the value of *n* so that Plan B is better for Tom.

Tools
1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model

Application

Steps
1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

The perimeter of a rectangular
sign is not to exceed 50 ft.
The length is to be twice the width.
What widths will meet these conditions?

Tools
1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model